

PRELIMINARY SATELLITE ORBIT DETERMINATIONS USING A NEW SYSTEM FOR GEO-REFERENCING

R. Vilhena de Moraes^{1,3a}, H. Koiti Kuga^{1b}, P. Kaufmann^{2c}, P. L. Kaufmann^{3d} and S. V. D. Pamboukian^{2e}
¹INPE, São José dos Campos -SP, Brazil; ²U. P. MACKENZIE-SP, Brazil, ³UNIFESP, São José dos Campos-SP, Brazil;

e-mail: ^arodolpho.vilhena@gmail.com, ^bhelio.kuga@dem.inpe.br, ^cpierekau@gmail.com, ^dsergio.pamboukian@gmail.com

1. Introduction

The orbit of an artificial satellite carrying a radio signal transponder is determined using a new geo-referencing system. It utilizes four ground-based reference stations (see Figure 1), synchronized in time, installed at well known geodetic coordinates, and a repeater in space. It is based in coded time signal propagation times from one base to the others bounced back by the transponder. The process corrects added up time delays due to signal transit times in the transponder, and for delays due to propagation paths and electronics at distinct ends. We examine the process application for an artificial satellite carrying transponder, the orbit of which we want to determine. The influence of harmonics of high order and degree due to the non-uniform distribution of the Earth's mass are considered in orbit determination. The paper describes aspects of the new geo-referencing system and the dynamic modeling used in the orbit determination system. The specific uncertainties of measurements arising from this system are properly taken into account. A simulation performed with four ground bases located in Brazil and low altitude transponder proved the system concept. Results show that transponder position accuracies of less than 1 meter are attainable using standard good quality synchronized clocks. We extend this simulation to orbit determination of satellites. The major advantages and drawbacks of such system are discussed.

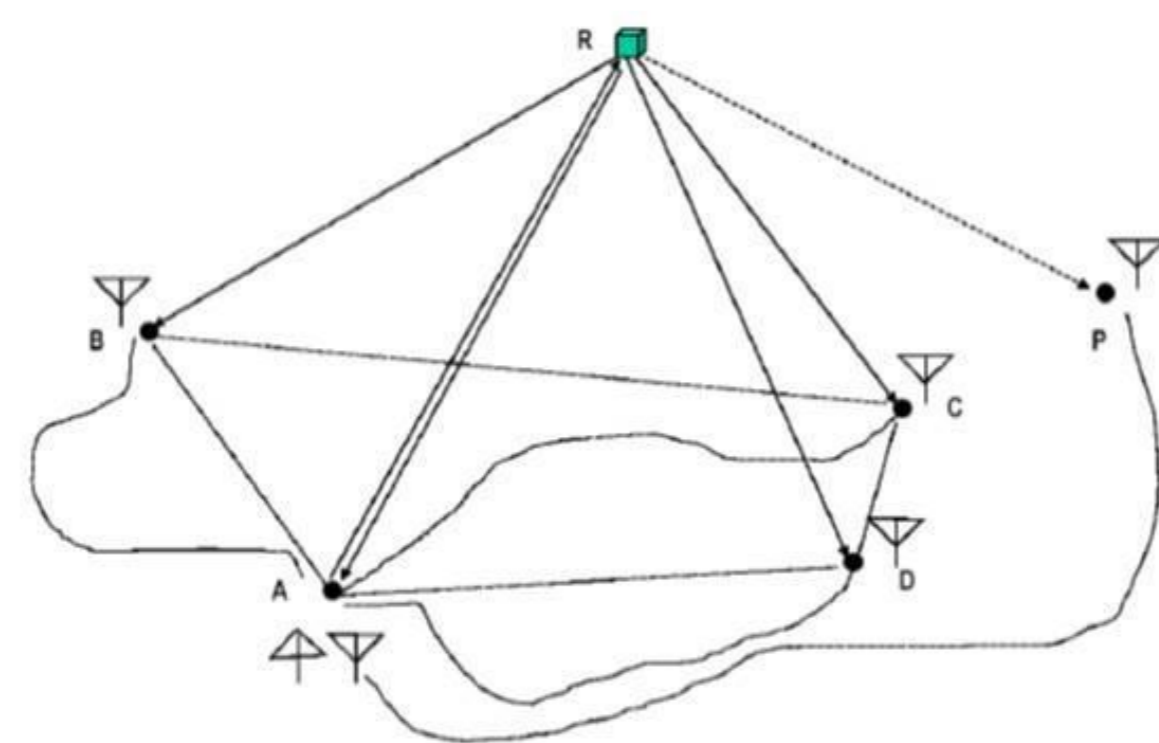


Figure 1- The four ground based geodetic reference bases, A,B,C and D, the repeater in the sky R, and a remote site P. Time signals are emitted by one reference base (A), retransmitted by R and received by all ground bases and the target where time differences are measured

2. Position Measurements with Geolocal

Let us consider four references ground bases A, B, C, D at known geodesic positions coordinates, synchronized among themselves and a repeater R in an artificial satellite. The coordinates (X_R, Y_R, Z_R) of the repeater are defined in the reference system illustrated in Figure 2. The ranging measurements obtained at a given instant can be written as

$$AR = \sqrt{(x_A - x_R)^2 + (y_A - y_R)^2 + (z_A - z_R)^2} = (\Delta t_A - \delta_{AR} - \delta_{AR}) \left(\frac{c}{2} \right) - \Delta_{pdAR}$$

$$BR = \sqrt{(x_B - x_R)^2 + (y_B - y_R)^2 + (z_B - z_R)^2} = (\Delta t_B - \delta_{BR} - \delta_{BR}) c - \Delta_{pdBR} - \Delta_{pdAR}$$

$$CR = \sqrt{(x_C - x_R)^2 + (y_C - y_R)^2 + (z_C - z_R)^2} = (\Delta t_C - \delta_{CR} - \delta_{CR}) c - \Delta_{pdCR} - \Delta_{pdAR}$$

$$DR = \sqrt{(x_D - x_R)^2 + (y_D - y_R)^2 + (z_D - z_R)^2} = (\Delta t_D - \delta_{DR} - \delta_{DR}) c - \Delta_{pdDR} - \Delta_{pdAR}$$

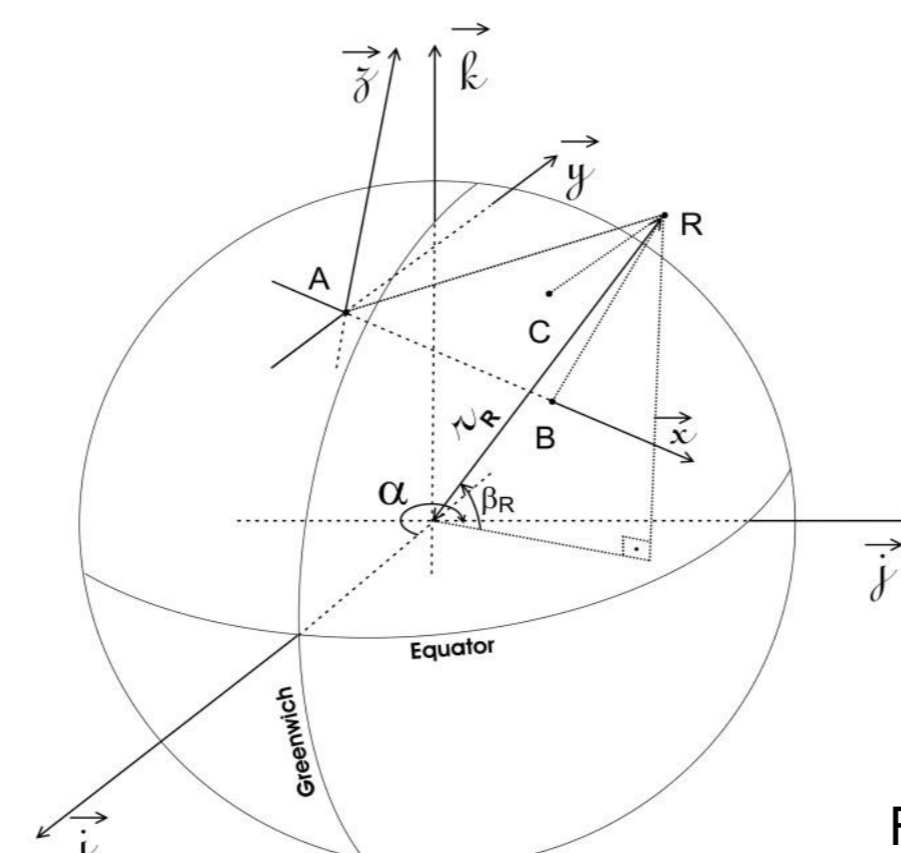


Figure 2

where $AR, BR, CR,$ and DR are the distances of bases A, B, C and D to the repeater R, respectively, expressed as a function of time variations caused by the signal transit at the repeater δR , to be determined, and corrected for the respective propagation path delays $\Delta_{pdAR}, \Delta_{pdBR}, \Delta_{pdCR}$ and Δ_{pdDR} . $\Delta t_A, \Delta t_B, \Delta t_C$ and Δt_D are the time differences effectively measured at bases A, B, C and D, respectively, with respect to their clocks; $\delta \Delta t$ is the time variation due to signal transit in circuits and cables when transmitted from base A, previously measured and known; $\delta_{AR}, \delta_{BR}, \delta_{CR}$ and δ_{DR} are the time variations due to the coded time signal transits on circuits and cables when received at bases A, B, C, and D, respectively, and c is the speed in free space of the electromagnetic waves that transport the coded time signal.

The pair of systems of equations to obtain the repeaters' coordinates in relation to bases A,B,C and A,B,D (or B,C,D and A,C,D and so on) allow us to determine the discrepancy between the repeater's positions caused by the propagation path delays, transit times δ_R at the repeater and delays at the bases. Therefore all delay parameters become negligible by minimizing the following expression:

$$f(\delta_R) = \|[X_R(\delta_R), Y_R(\delta_R), Z_R(\delta_R)] - [X'_R(\delta_R), Y'_R(\delta_R), Z'_R(\delta_R)]\|^2 = [X_R(\delta_R) - X'_R(\delta_R)]^2 + [Y_R(\delta_R) - Y'_R(\delta_R)]^2 + [Z_R(\delta_R) - Z'_R(\delta_R)]^2$$

In this procedure, δ_R actually include the transit times as well the path delays, and other instrumental delays. The closest value found for δ_R for every time signal interaction at the repeater corresponds to the minimum value obtained for $f(\delta_R)$ with varying values of δ_R . This can be found by a number of well known numerical calculation methods, as it was demonstrated by the simulations later in this paper. The accuracy of δ_R determinations will be in the limit of the instrumental accuracies utilized for the coordinate's determination.

3. Preliminary (Quick Look) Orbit Determination

The Geolocal system will provide locations of repeaters and target by using geometry, algorithms, and high-technology clock synchronization, as described in Kaufmann et al. (2012; 2014). The accuracy of the system has been reported here and it is well known that orders of meters accuracy are possible. In this context, one describes a quick look method to determine preliminarily the orbit of a satellite using a time sequence of 3 position vectors computed by the Geolocal system. Assume that Geolocal yields 3 position vectors $\vec{r}_i, i=1,2,3$ at 3 different instants. Some computation has to be made to transform those coordinates to inertial system of reference, say J2000.

Then we can compute the three velocities at these instants by using the well known formula

$$\vec{v}_i = L \left(\frac{\vec{r}_i}{r_i} + \vec{S} \right)$$

$$\vec{S} = (r_2 - r_3) \vec{r}_1 + (r_3 - r_1) \vec{r}_2 + (r_1 - r_2) \vec{r}_3$$

$$\vec{B} = \vec{D} \wedge \vec{r}_i \quad ; \quad L = (\mu / \Delta D)^{1/2}$$

$$\vec{N} = r_2 (\vec{r}_1 \wedge \vec{r}_2) + r_1 (\vec{r}_2 \wedge \vec{r}_3) + r_3 (\vec{r}_3 \wedge \vec{r}_1)$$

$$\vec{D} = \vec{r}_1 \wedge \vec{r}_2 + \vec{r}_2 \wedge \vec{r}_3 + \vec{r}_3 \wedge \vec{r}_1$$

If velocity vectors are recovered at $i=1,2,3$ then the preliminary orbit at such instants are promptly available, and one can use the analytical methods to propagate quickly the orbit anywhere in time.

4. Simulations and Tests of Quick Look Orbit Determination

The Figure 3 shows the geometry between the 4 repeaters and the orbit, used by Geolocal to compute the position vector of the orbit in successive instants. The clocks (master and the others) are assumed being at the station locations specified in Table 1.



Figure 3 – Geometry between the stations and orbit.

Name	Longitude (°)	Latitude (°)	Altitude (m)
São Paulo	-46.64	-23.63	700
Vitoria	-40.30	-20.31	3
Brasília	-47.91	-15.72	1100
Campo Grande	-54.65	-19.75	600

Table 1 – Simulated locations of the ground stations of Geolocal system.

Date UTC	X (m)	Y (m)	Z (m)
12.02.2015 22:00:00	669801.8	676290.9	-3667946.8
12.02.2015 22:00:10	659801.8	6799383.8	-3611700.7
12.02.2015 22:00:20	69746.5	6831889.0	-3551418.8
12.02.2015 22:00:30	579638.5	6863803.7	-3498274.9
12.02.2015 22:00:40	549480.3	6895125.1	-3441105.0
12.02.2015 22:00:50	519274.7	6925880.5	-3383636.9
12.02.2015 22:01:00	489024.2	6955977.1	-3325875.7
12.02.2015 22:01:10	458731.4	6985502.4	-3267826.3
12.02.2015 22:01:20	428398.9	7014423.8	-3209493.8
12.02.2015 22:01:30	398029.4	7042738.8	-3150882.2
12.02.2015 22:01:40	367625.5	7070444.9	-3091999.6
12.02.2015 22:01:50	337189.8	7097539.6	-3032848.0
12.02.2015 22:02:00	306734.9	7124020.8	-2973433.7
12.02.2015 22:02:10	276233.5	7149883.9	-2913761.7
12.02.2015 22:02:20	245718.2	7175132.8	-285387.2
12.02.2015 22:02:30	215181.7	7199759.3	-2793665.4
12.02.2015 22:02:40	184626.6	7223763.2	-2733254.4
12.02.2015 22:02:50	154055.5	7247142.4	-2672600.6
12.02.2015 22:03:00	123471.0	7269894.9	-2611718.2

Table 2 – Orbit position vectors: 3 minutes of data which was provided at every 10s.

Time spacing (s)	Radial error (m)	Normal error (m)	In-track error (m)	Total error (m)	Initial velocity error (m/s)	Pointing error (°)
10	72.1	16712.2	-22667.1	28323.2	1.44	0.21

Table 3 – Errors after one orbit period of prediction (time spacing 10s)

Time spacing (s)	Radial error (m)	Normal error (m)	In-track error (m)	Total error (m)	Initial velocity error (m/s)	Pointing error (°)
10	72.10	16712.30	-22667.10	28323.20	1.44	0.21
20	612.80	16812.50	-95288.90	96762.70	1.52	0.72
30	1831.40	16911.00	-167642.50	168503.30	1.60	1.25
60	9551.00	17195.70	-384523.70	385026.50	1.83	2.86
80	18079.80	17376.50	-528916.70	529510.80	1.98	3.93

Table 4 – Errors after one orbit period of prediction for several time spacings

Kaufmann et al. (2014, 2012) simulated typical errors of in the order of few meters on the repeater's (or satellite) positions, assuming the four reference clock synchronization uncertainty of ± 5 ns spread around zero with a Gaussian profile. In this work one assumes regular meter level achieved by Geolocal when computing the position vectors. Therefore a quite straightforward reasoning is that a Gaussian profile could also be adopted for the position errors as provided by Geolocal system.

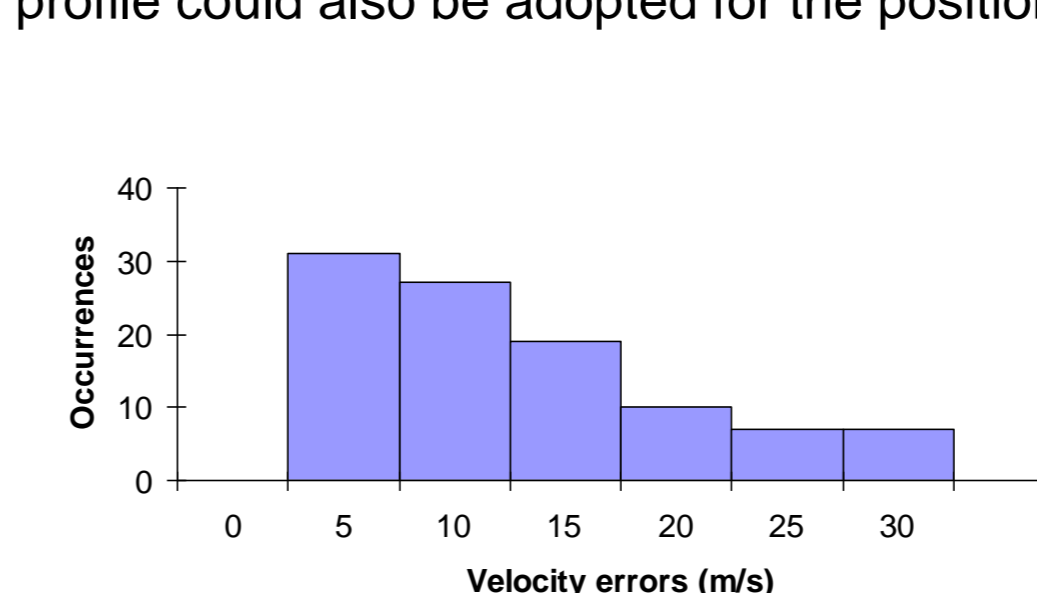


Figure 4 – Error distribution of initial velocity and pointing for 10s time spacing

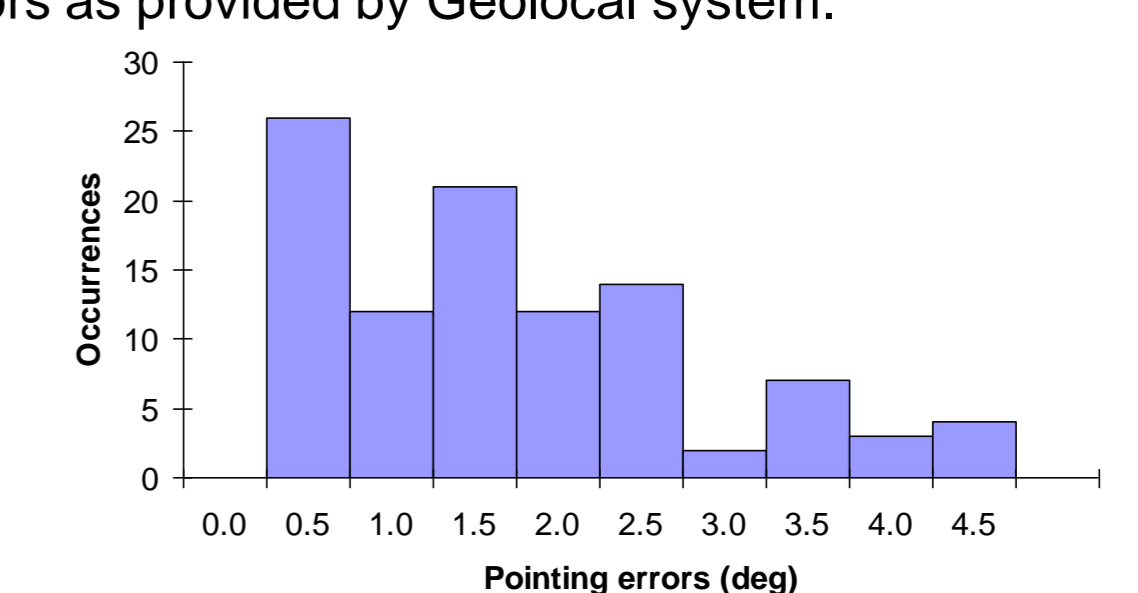


Figure 5 – Error distribution of initial velocity and pointing for 60s time spacing

6. Conclusions

The work has described a method for quick look orbit determination from 3 position vectors generated by the Geolocal system. A careful analysis has shown that the choice of time spacing between the position vectors is paramount to the performance of the procedure.

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