









PDOP SIMULATIONS OF GEOLOCAL: A GEO-REFERENCING SYSTEM INDEPENDENT FROM GNSS SYSTEMS

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1. Introduction

Geolocal is a navigation system developed by researchers from Universidade Mackenzie, in Brazil. It is based on an "inverted-GNSS" geometry, as shown in Figure 1, that has at least 4 base stations on the ground, A, B, C and D, at known geodesic position coordinates, synchronized among themselves, a repeater R in space, that can be a spacecraft, a balloon or a satellite, and a target P on the ground (KALFMANN et al, 2014).

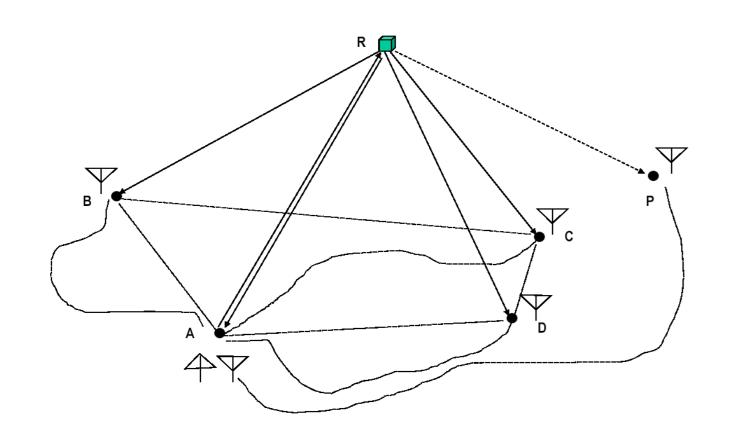


Figure 1- The four ground based geodetic reference bases, A,B,C and D, the repeater in space R, and a remote target P. Time signals are emitted by one reference base (A), retransmitted by R and received by all ground bases and the target where time differences are measured (KALFMANN et al, 2014).

To establish the position of the repeater, a base chosen as reference base, in this case, base A, sends a radio signal that is received by the repeater, and retransmitted to all bases, producing at least 4 ranging measurements that allow us to calculate the distances AR, BR, CR and DR using the equations bellow. With these distances, it is possible to find the position of the repeater by triangulation.

$$AR(\delta_{R}, \Delta_{pdAR}) = (\Delta t_{A} - \delta_{At} - \delta_{Ar} - \delta_{R})(^{C}/_{2}) - \Delta_{pdAR}$$

$$BR(\delta_{R}, \Delta_{pdAR}, \Delta_{pdBR}) = (\Delta t_{B} - \delta_{At} - \delta_{Br} - \delta_{R})c - AR(\delta_{R}) - \Delta_{pdBR} - \Delta_{pdAR}$$

$$CR(\delta_{R}, \Delta_{pdAR}, \Delta_{pdCR}) = (\Delta t_{C} - \delta_{At} - \delta_{Cr} - \delta_{R})c - AR(\delta_{R}) - \Delta_{pdCR} - \Delta_{pdAR}$$

$$DR(\delta_{R}, \Delta_{pdAR}, \Delta_{pdDR}) = (\Delta t_{D} - \delta_{At} - \delta_{Dr} - \delta_{R})c - AR(\delta_{R}) - \Delta_{pdDR} - \Delta_{pdAR}$$

$$PR(\delta_{R}, \Delta_{pdAR}, \Delta_{pdPR}) = (\Delta t_{P} - \delta_{At} - \delta_{Pr} - \delta_{R})c - AR(\delta_{R}) - \Delta_{pdPR} - \Delta_{pdAR}$$

These equations depend on the repeater's delay δ_R and on the propagation delays Δ_{pdAR} , Δ_{pdBR} , Δ_{pdBR} , Δ_{pdCR} , Δ_{pdDR} and Δ_{pdPR} . The values Δ_{tA} , Δ_{tB} , Δ_{tC} , Δ_{tD} and Δ_{tP} are the time differences measured at each base and target, when comparing their clock with the signal received. The value δ_{At} is the known time variation due to the signal transit through circuits and cables in the transmitting base; δ_{Ar} , δ_{Br} , δ_{Cr} , δ_{Dr} e δ_{Pr} are the know time variation due to the signal transit through circuits and cables in the receiving bases and the target.

2. PDOP simulations with 4 and 5 fixed bases

PDOP (Position Dilution of Precision) is a factor that indicates how diluted is the precision of the position, in other words, based on the geometry of the system, how the position error of the repeater increases from the error of the distances AR, BR, CR and DR. The ideal PDOP value for most applications is 1, but 4 is considered acceptable.

To calculate PDOP in the ECEF (Earth Centered – Earth Fixed) system, it is used the A Matrix and the equations below (DANNA, 1996).

$$A = \begin{bmatrix} \frac{Ax - Rx}{AR} & \frac{Ay - Ry}{AR} & \frac{Az - Rz}{AR} & -1 \\ \frac{Bx - Rx}{BR} & \frac{By - Ry}{BR} & \frac{Bz - Rz}{BR} & -1 \\ \frac{Cx - Rx}{CR} & \frac{Cy - Ry}{CR} & \frac{Cz - Rz}{CR} & -1 \\ \frac{Dx - Rx}{DR} & \frac{Dy - Ry}{DR} & \frac{Dz - Rz}{DR} & -1 \end{bmatrix}$$

$$AR = \sqrt{(Ax - Rx)^2 + (Ay - Ry)^2 + (Az - Rz)^2}$$

$$BR = \sqrt{(Bx - Rx)^2 + (By - Ry)^2 + (Bz - Rz)^2}$$

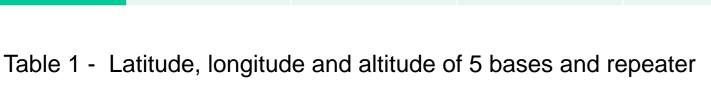
$$CR = \sqrt{(Cx - Rx)^2 + (Cy - Ry)^2 + (Cz - Rz)^2}$$

$$DR = \sqrt{(Dx - Rx)^2 + (Dy - Ry)^2 + (Dz - Rz)^2}$$

$$PDOP = \sqrt{P_{0,0} + P_{1,1} + P_{2,2}}$$

As an example, it was done some simulations with 5 bases A, B, C, D and E and a repeater R in 5 cities of Sao Paulo state, Brazil, in the coordinates show in Table 1 and illustrated in the Figure 2. PDOP results, elevation at each base, and average elevation for 5 combinations of 4 bases and 1 combination of 5 bases are shown in Table 2.

City	Element	Latitude	Longitude	Altitude
Rio Claro	Base A	- 22° 24' 48''	- 47° 34' 11''	592 m
Piracicaba	Base B	- 22° 43′ 30′′	- 47° 38' 51''	524 m
Botucatu	Base C	- 22° 53' 25"	- 48° 27' 19''	828 m
Campinas	Base D	- 22° 54' 23''	- 47° 03' 42''	677 m
Itu	Base E	- 23° 15' 57''	- 47° 17' 57''	577 m
Piracicaba	Repeater R	- 22° 43′ 30′′	- 47° 38' 51''	variable



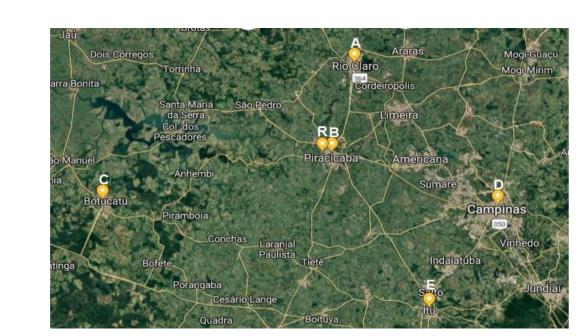


Figure 2 – position of the 5 bases, A, B, C and D, and repeater R (adapted from Google Maps)

Altitude	(m)	10000	20000	30000	40000	50000	60000	Altitude (m)		10000	20000	30000	40000	50000	60000
Combination ABCD	PDOP	1,86	2,16	2,57	3,09	3,72	4,45	Combination ACDE	PDOP	29,71	16,93	13,86	13,25	13,70	14,78
	Elevation at A	14,86°	28,68°	39,63°	47,95°	54,25°	59,07°		Elevation at A	14,86°	28,68°	39,63°	47,95°	54,25°	59,07°
	Elevation at B	90,00°	90,00°	90,00°	90,00°	90,00°	90,00°		Elevation at C	6,16°	12,70°	18,91°	24,69°	29,97°	34,74°
	Elevation at C	6,16°	12,70°	18,91°	24,69°	29,97°	34,74°		Elevation at D	8,36°	16,92°	24,77°	31,72°	37,77°	42,96°
	Elevation at D	8,36°	16,92°	24,77°	31,72°	37,77°	42,96°		Elevation at E	7,69°	15,54°	22,83°	29,40°	35,22°	40,30°
	Average elevation	9,79°	19,43°	27,77°	34,79°	40,66°	45,59°		Average elevation	9,27°	18,46°	26,54°	33,44°	39,30°	44,27°
	PDOP	1,85	2,19	2,67	3,28	4,02		4,90 59,07° 90,00° Combination BCDE 34,74° 40,30° 44,70° 8,76 59,07° 90,00° Combination ABCDE 42,96° 40,30°	PDOP	3.02	3,28	3,65	4,12	4,70	5,40
	Elevation at A	14,86°	28,68°	39,63°	47,95°	54,25°			Elevation at B	90,00°	90,00°	90,00°	90,00°	90,00°	90,00°
		90,00°	90,00°	90,00°	90,00°	90,00°			Elevation at C	6,16°	12,70°	18,91°	24,69°	29,97°	34,74°
Combination ABCE	Elevation at B	,	,	,			·		Elevation at D	8,36°	16,92°	24,77°	31,72°	37,77°	42,96°
	Elevation at C	6,16°	12,70°	18,91°	24,69°	29,97°			Elevation at E	7,69°	15,54°	22,83°	29,40°	35,22°	40,30°
	Elevation at E	7,69°	15,54°	22,83°	29,40°	35,22°	40,30°		Average elevation	7,40°	15,05°	22,17°	28,60°	34,32°	39,33°
	Average elevation	9,57°	18,97°	27,12°	34,01°	39,81°	44,70°		PDOP	1,72	2,07	2,46	2,93	3,45	3,99
	PDOP	3,47	4,04	4,84	5,89	7,19	8,76		Elevation at A	14,86°	28,68°	39,63°	47,95°	54,25°	59,07°
Combination ABDE	Elevation at A	14,86°	28,68°	39,63°	47,95°	54,25°	59,07°		Elevation at B	90,00°	90,00°	90,00°	90,00°	90,00°	90,00°
	Elevation at B	90,00°	90,00°	90,00°	90,00°	90,00°	90,00°		Elevation at C	6,16°	12,70°	18,91°	24,69°	29,97°	34,74°
	Elevation at D	8,36°	16,92°	24,77°	31,72°	37,77°	42,96°		Elevation at D	8,36°	16,92°	24,77°	31,72°	37,77°	42,96°
	Elevation at E	7,69°	15,54°	22,83°	29,40°	35,22°	40,30°		Elevation at E	7,69°	15,54°	22,83°	29,40°	35,22°	40,30°
	Average elevation	10,30°	20,38°	29,08°	36,36°	42,41°	47,44°		Average elevation	9,27°	18,46°	26,54°	33,44°	39,30°	44,27°

Table 2 - PDOP and elevations simulating 5 combination of 4 fixed bases and 1 combination of 5 fixed bases for each repeater

3. PDOP simulations with 12 fixed bases

Other simulations were done with 12 fixed bases are show in Table 3 and illustrated in the Figure 3. The results are shown in Table 4.

City	Element	Latitude	Longitude	Altitude	
Rio Claro	Base A	- 22° 24' 48''	- 47° 34' 11''	592 m	
Piracicaba	Base B	- 22° 43′ 30′′	- 47° 38' 51''	524 m	
Botucatu	Base C	- 22° 53' 25''	- 48° 27' 19''	828 m	
Campinas	Base D	- 22° 54' 23''	- 47° 03' 42''	677 m	
Itu	Base E	- 23° 15' 57''	- 47° 17' 57''	577 m	
Jaú	Base F	-22° 17' 47''	-48° 33' 28''	522 m	
Brotas	Base G	-22° 17' 03''	48° 07' 36''	647 m	
Santa Maria da Serra	Base H	-22° 34' 02''	-48° 09' 38''	495 m	
Conchas	Base I	-23° 00' 55''	-48° 00' 38''	503 m	
Boituva	Base J	-23° 17' 00''	-47° 40' 20''	637 m	
Mogi Mirim	Base K	-22° 25' 55''	-46° 57' 30''	617 m	
Bofete	Base L	-23° 06' 08''	-48° 15' 28''	576 m	
Piracicaba	Repeater R1	- 22° 43' 30''	- 47° 38' 51''	variable	
Limeira	Repeater R2	-22° 33' 52''	-47° 24' 01''	variable	
Charqueada	Repeater R3	-22° 30' 39''	-47° 46' 40''	variable	
Anhembi	Repeater R4	-22° 47' 22''	-48° 07' 38''	variable	

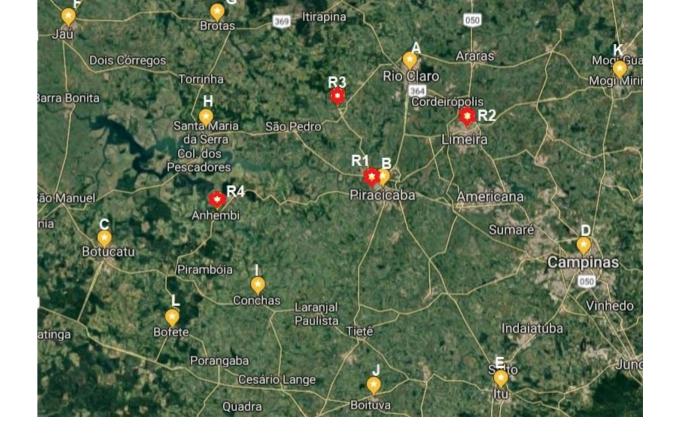


Table 3 - Latitude, longitude and altitude of 12 bases and 4 repeaters

Figure 3 – position of the 12 bases and the 4 repeaters (adapted from Google Maps)

Altitude (m)		10000	20000	30000	40000	50000	60000	Altitud	e (m)	10000	20000	30000	40000	50000	60000
	PDOP	1,38	1,56	1,75	1,96	2,20	2,48		PDOP	3,66	2,30	2,07	2,12	2,28	2,51
	Elevation at A	14,86°	28,68°	39,63°	47,95°	54,25°	59,07°		Elevation at A	21,40°	38,94°	50,74°	58,60°	64,02°	67,92°
	Elevation at B	90,00°	90,00°	90,00°	90,00°	90,00°	90,00°		Elevation at B	19,17°	35,52°	47,19°	55,31°	61,07°	65,29°
	Elevation at C	6,16°	12,70°	18,91°	24,69°	29,97°	34,74°		Elevation at C	6,43°	13,25°	19,69°	25,65°	31,06°	35,92°
	Elevation at D	8,36°	16,92°	24,77°	31,72°	37,77°	42,96°		Elevation at D	6,21°	12,70°	18,86°	24,60°	29,85°	34,59°
	Elevation at E	7,69°	15,54°	22,83°	29,40°	35,22°	40,30°		Elevation at E	5,55°	11,31°	16,84°	22,06°	26,92°	31,38°
Domanton D1	Elevation at F	5,15°	10,49°	15,65°	20,55°	25,14°	29,41°	Repeater R3	Elevation at F	6,45°	13,07°	19,35°	25,17°	30,47°	35,25
Repeater R1	Elevation at G	7,67°	15,56°	22,88°	29,48°	35,32°	40,41°	Repeater K3	Elevation at G	12,04°	23,79°	33,75°	41,83°	48,28°	53,43°
	Elevation at H	9,70°	19,32°	27,92°	35,33°	41,59°	46,83°		Elevation at H	13,40°	26,03°	36,44°	44,65°	51,04°	56,05°
	Elevation at I	10,91°	21,58°	30,88°	38,66°	45,05°	50,27°		Elevation at I	8,88°	17,76°	25,84°	32,94°	39,06°	44,26
	Elevation at J	8,60°	17,35°	25,33°	32,38°	38,47°	43,67°		Elevation at J	6,19°	12,64°	18,76°	24,47°	29,69°	34,42
	Elevation at K	6,86°	13,94°	20,61°	26,73°	32,25°	37,17°		Elevation at K	6,31°	12,86°	19,07°	24,85°	30,12°	34,880
	Elevation at L	7,13°	14,45°	21,31°	27,57°	33,19°	38,16°		Elevation at L	6,55°	13,31°	19,70°	25,62°	30,99°	35,81°
	Ave. elevation	8,46°	16,95°	24,61°	31,32°	37,11°	42,09°		Ave. elevation	9,88°	19,27°	27,19°	33,81°	39,38°	44,10°
	PDOP	3,87	2,44	2,23	2,30	2,51	2,78		PDOP	3,95	2,51	2,29	2,35	2,53	2,78
	Elevation at A	21,25°	38,72°	50,52°	58,40°	63,85°	67,77°		Elevation at A	7,56°	15,29°	22,49°	29,00°	34,78°	39,84°
	Elevation at B	16,98°	32,09°	43,48°	51,76°	57,82°	62,35°		Elevation at B	10,77°	21,33°	30,57°	38,32°	44,71°	49,94°
	Elevation at C	4,59°	9,51°	14,29°	18,87°	23,21°	27,27°		Elevation at C	14,48°	28,35°	39,36°	47,74°	54,08°	58,94°
	Elevation at D	10,27°	20,57°	29,64°	37,32°	43,70°	48,95°		Elevation at D	4,83°	9,94°	14,88°	19,59°	24,04°	28,20°
	Elevation at E	6,85°	13,90°	20,53°	26,63°	32,14°	37,04°		Elevation at E	5,38°	10,98°	16,37°	21,47°	26,22°	30,62
D 4 D2	Elevation at F	4,41°	9,00°	13,47°	17,77°	21,87°	25,74°	Donoston D4	Elevation at F	7,67°	15,46°	22,70°	29,24°	35,03°	40,10°
Repeater R2	Elevation at G	6,58°	13,42°	19,88°	25,85°	31,26°	36,11°	14° 47° 01°	Elevation at G	9,48°	19,05°	27,62°	35,03°	41,30°	46,55
	Elevation at H	6,93°	13,98°	20,63°	26,73°	32,24°	37,14°		Elevation at H	20,92°	38,09°	49,83°	57,75°	63,26°	67,24°
	Elevation at I	6,75°	13,66°	20,17°	26,17°	31,61°	36,47°		Elevation at I	18,89°	35,07°	46,70°	54,85°	60,65°	64,91°
	Elevation at J	6,33°	12,91°	19,15°	24,94°	30,24°	35,01°		Elevation at J	7,42°	15,05°	22,17°	28,63°	34,37°	39,42°
	Elevation at K	11,10°	22,05°	31,53°	39,41°	45,83°	51,05°		Elevation at K	4,24°	8,70°	13,04°	17,24°	21,24°	25,04°
	Elevation at L	5,06°	10,34°	15,44°	20,29°	24,85°	29,10°		Elevation at L	14,23°	27,57°	38,32°	46,62°	52,97°	57,87
	Ave. elevation	8,93°	17,51°	24,89°	31,18°	36,55°	41,17°		Ave. elevation	10,49°	20,41°	28,67°	35,46°	41,05°	45,72°

Table 4 - PDOP and elevations of 12 bases and 4 repeaters

4. PDOP analysis

In the simulations using 4 bases, shown in Figure 2, the combination ACDE, that is not considering base B, resulted in higher values of PDOP, and from these values, the lowest happened at 35° elevation. For 4 bases, the best distribution happens when the repeater is over one of the base, and the other bases are evenly distributed around, and that happened in the combination ABCD and ABCE. When a fifth base is added to the system, the PDOP factor decreases. Considering a cutoff angle of 15°, the best elevation for this geometry is 30000 meters.

The simulation with 12 bases, shown in Figure 3, brought PDOP factors within acceptable values, especially for repeater R1, where these values are near the ideal. In R1, the PDOP value increases with altitude, however the altitudes of 10000 and 20000 meters are not recommended because there are several occurrences of elevations bellow 15°, so the altitude of 30000 meters are the best for this configuration. It is possible to use repeaters of lower altitude, but the bases would need to be closer from each other.

Regarding the repeaters R2, R3, and R4, the PDOP value decreases until 30000 meters of altitude and then increases again. The repeater R2 has PDOP 2.23 at 30000 meters, but there are 2 values bellow 15° (bases C and F) that could be disconsidered in the PDOP calculation. Doing this, the PDOP drops to 2.51 and the repeater at 40000 meters of altitude would have a PDOP lower than 30000 meters. The repeater R3 doesn't have elevation lower than 15° at 30000 meters and R4 has 2 (base D and K). Disconsidering these bases, the PDOP drops to 2.46, that is greater than 2.35 at 40000 meters.

4. Conclusion

PDOP is a factor that indicates how the distribution of the bases affects the accuracy of the measurements, and it could be used as a geometry parameter of the Geolocal system. For 4 bases, the lowest PDOP happens when the repeater is located over one of the bases, and, as the number of bases increases, the PDOP value decreases.

The best altitude of repeater depends on the distance from the bases, and in order to use a repeater in lower altitude, it is necessary to use bases closer to each other.

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