

PDOP SIMULATIONS OF GEOLOCAL: A GEO-REFERENCING SYSTEM INDEPENDENT FROM GNSS SYSTEMS

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1. Introduction

Geolocal is a navigation system developed by researchers from Universidade Mackenzie, in Brazil. It is based on an "inverted-GNSS" geometry, as shown in Figure 1, that has at least 4 base stations on the ground, A, B, C and D, at known geodesic position coordinates, synchronized among themselves, a repeater R in space, that can be a spacecraft, a balloon or a satellite, and a target P on the ground (KALFMANN et al, 2014).

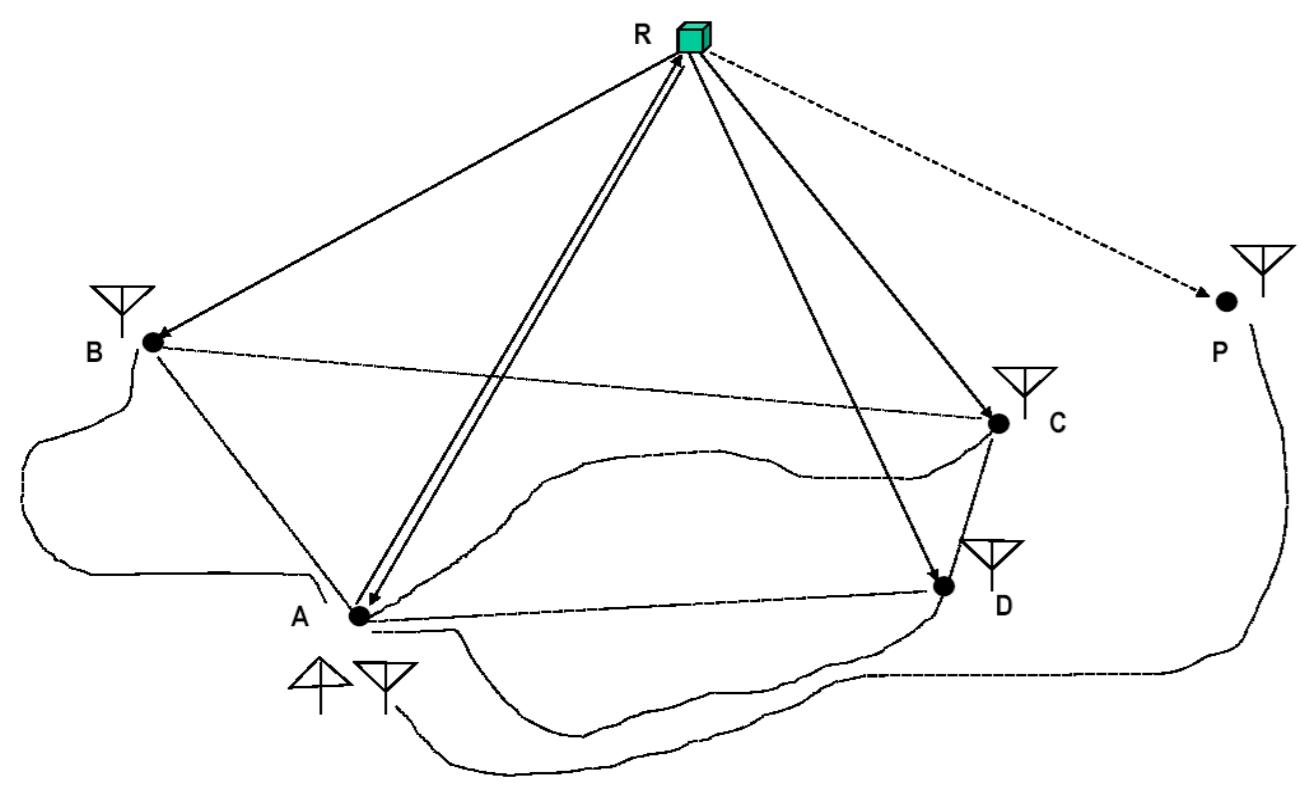


Figure 1- The four ground based geodetic reference bases, A,B,C and D, the repeater in space R, and a remote target P. Time signals are emitted by one reference base (A), retransmitted by R and received by all ground bases and the target where time differences are measured (KALFMANN et al, 2014).

To establish the position of the repeater, a base chosen as reference base, in this case, base A, sends a radio signal that is received by the repeater, and retransmitted to all bases, producing at least 4 ranging measurements that allow us to calculate the distances AR, BR, CR and DR using the equations below. With these distances, it is possible to find the position of the repeater by triangulation.

$$AR(\delta_R, \Delta_{pdAR}) = (\Delta t_A - \delta_{At} - \delta_{Ar} - \delta_R)(c/2) - \Delta_{pdAR}$$

$$BR(\delta_R, \Delta_{pdBR}) = (\Delta t_B - \delta_{Bt} - \delta_{Br} - \delta_R)c - AR(\delta_R) - \Delta_{pdBR} - \Delta_{pdAR}$$

$$CR(\delta_R, \Delta_{pdCR}) = (\Delta t_C - \delta_{Ct} - \delta_{Cr} - \delta_R)c - AR(\delta_R) - \Delta_{pdCR} - \Delta_{pdAR}$$

$$DR(\delta_R, \Delta_{pdDR}) = (\Delta t_D - \delta_{Dt} - \delta_{Dr} - \delta_R)c - AR(\delta_R) - \Delta_{pdDR} - \Delta_{pdAR}$$

$$PR(\delta_R, \Delta_{pdPR}) = (\Delta t_P - \delta_{Pt} - \delta_{Pr} - \delta_R)c - AR(\delta_R) - \Delta_{pdPR} - \Delta_{pdAR}$$

These equations depend on the repeater's delay δ_R and on the propagation delays Δ_{pdAR} , Δ_{pdBR} , Δ_{pdCR} , Δ_{pdDR} and Δ_{pdPR} . The values Δ_{tA} , Δ_{tB} , Δ_{tC} , Δ_{tD} and Δ_{tP} are the time differences measured at each base and target, when comparing their clock with the signal received. The value δ_{At} is the known time variation due to the signal transit through circuits and cables in the transmitting base; δ_{Ar} , δ_{Br} , δ_{Cr} , δ_{Dr} e δ_{Pr} are the know time variation due to the signal transit through circuits and cables in the receiving bases and the target.

2. PDOP simulations with 4 and 5 fixed bases

PDOP (Position Dilution of Precision) is a factor that indicates how diluted is the precision of the position, in other words, based on the geometry of the system, how the position error of the repeater increases from the error of the distances AR, BR, CR and DR. The ideal PDOP value for most applications is 1, but 4 is considered acceptable.

To calculate PDOP in the ECEF (Earth Centered – Earth Fixed) system, it is used the A Matrix and the equations below (DANNA, 1996).

$$A = \begin{bmatrix} Ax - Rx & Ay - Ry & Az - Rz \\ AR & BR & CR \\ Bx - Rx & By - Ry & Bz - Rz \\ BR & BR & BR \\ Cx - Rx & Cy - Ry & Cz - Rz \\ CR & CR & CR \\ Dx - Rx & Dy - Ry & Dz - Rz \\ DR & DR & DR \end{bmatrix} \begin{matrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{matrix}$$

$$AR = \sqrt{(Ax - Rx)^2 + (Ay - Ry)^2 + (Az - Rz)^2}$$

$$BR = \sqrt{(Bx - Rx)^2 + (By - Ry)^2 + (Bz - Rz)^2}$$

$$CR = \sqrt{(Cx - Rx)^2 + (Cy - Ry)^2 + (Cz - Rz)^2}$$

$$DR = \sqrt{(Dx - Rx)^2 + (Dy - Ry)^2 + (Dz - Rz)^2}$$

$$P = (A^T \cdot A)^{-1}$$

$$PDOP = \sqrt{P_{0,0} + P_{1,1} + P_{2,2}}$$

As an example, it was done some simulations with 5 bases A, B, C, D and E and a repeater R in 5 cities of Sao Paulo state, Brazil, in the coordinates show in Table 1 and illustrated in the Figure 2. PDOP results, elevation at each base, and average elevation for 5 combinations of 4 bases and 1 combination of 5 bases are shown in Table 2.

City	Element	Latitude	Longitude	Altitude
Rio Claro	Base A	-22° 24' 48"	-47° 34' 11"	592 m
Piracicaba	Base B	-22° 43' 30"	-47° 38' 51"	524 m
Botucatu	Base C	-22° 53' 25"	-48° 27' 19"	828 m
Campinas	Base D	-22° 54' 23"	-47° 03' 42"	677 m
Ita	Base E	-23° 15' 57"	-47° 17' 57"	577 m
Piracicaba	Repeater R	-22° 43' 30"	-47° 38' 51"	variable

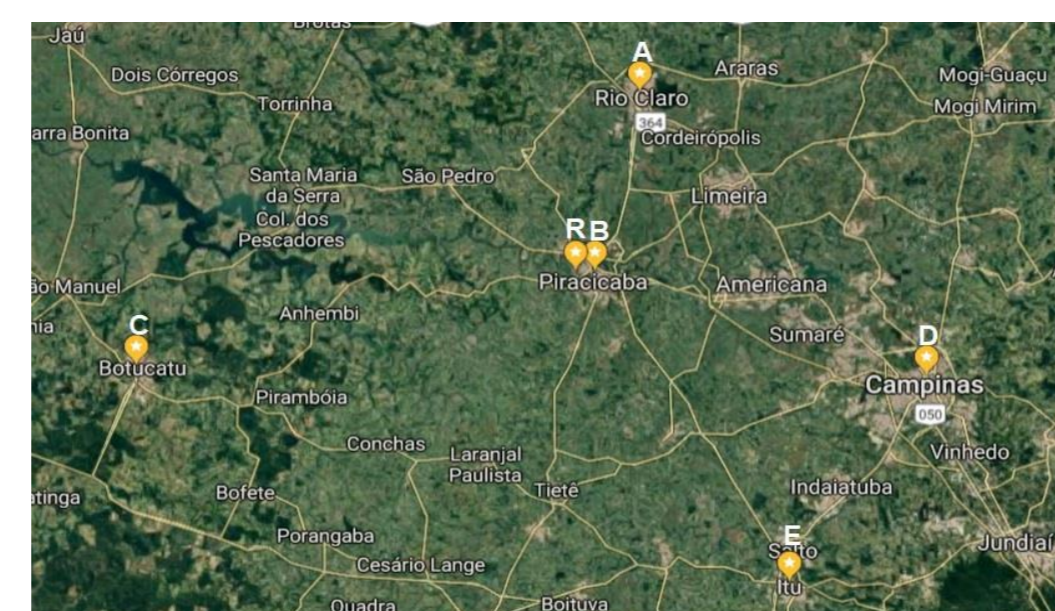


Figure 2 – position of the 5 bases, A, B, C and D, and repeater R (adapted from Google Maps)

Combination	Altitude (m)	PDOP					Average elevation
		10000	20000	30000	40000	50000	
Combination ABCD	PDOP	1.86	2.16	2.57	3.09	3.72	4.45
	Elevation at A	14.86°	28.68°	39.63°	47.95°	54.25°	59.07°
	Elevation at B	90.00°	90.00°	90.00°	90.00°	90.00°	90.00°
	Elevation at C	6.16°	12.70°	18.91°	24.69°	29.97°	34.74°
	Elevation at D	8.36°	16.92°	24.77°	31.72°	37.77°	42.96°
	Average elevation	9.79°	19.47°	27.77°	34.79°	40.66°	45.59°
Combination ABCE	PDOP	1.85	2.19	2.67	3.28	4.02	4.90
	Elevation at A	14.86°	28.68°	39.63°	47.95°	54.25°	59.07°
	Elevation at B	90.00°	90.00°	90.00°	90.00°	90.00°	90.00°
	Elevation at C	6.16°	12.70°	18.91°	24.69°	29.97°	34.74°
	Elevation at E	7.69°	15.54°	22.83°	29.40°	35.22°	40.30°
	Average elevation	9.57°	18.97°	27.12°	34.01°	39.81°	44.70°
Combination ABCDE	PDOP	3.47	4.04	4.84	5.89	7.19	8.76
	Elevation at A	14.86°	28.68°	39.63°	47.95°	54.25°	59.07°
	Elevation at B	90.00°	90.00°	90.00°	90.00°	90.00°	90.00°
	Elevation at C	6.16°	12.70°	18.91°	24.69°	29.97°	34.74°
	Elevation at D	8.36°	16.92°	24.77°	31.72°	37.77°	42.96°
	Elevation at E	7.69°	15.54°	22.83°	29.40°	35.22°	40.30°
Combination ACDE	PDOP	2.97	3.28	3.65	4.12	4.70	5.40
	Elevation at A	14.86°	28.68°	39.63°	47.95°	54.25°	59.07°
	Elevation at B	90.00°	90.00°	90.00°	90.00°	90.00°	90.00°
	Elevation at C	6.16°	12.70°	18.91°	24.69°	29.97°	34.74°
	Elevation at D	8.36°	16.92°	24.77°	31.72°	37.77°	42.96°
	Elevation at E	7.69°	15.54°	22.83°	29.40°	35.22°	40.30°
Combination BCDE	PDOP	1.72	2.07	2.46	2.93	3.45	3.99
	Elevation at A	14.86°	28.68°	39.63°	47.95°	54.25°	59.07°
	Elevation at B	90.00°	90.00°	90.00°	90.00°	90.00°	90.00°
	Elevation at C	6.16°	12.70°	18.91°	24.69°	29.97°	34.74°
	Elevation at D	8.36°	16.92°	24.77°	31.72°	37.77°	42.96°
	Elevation at E	7.69°	15.54°	22.83°	29.40°	35.22°	40.30°
Combination ABDE	PDOP	2.97	3.28	3.65	4.12	4.70	5.40
	Elevation at A	14.86°	28.68°	39.63°	47.95°	54.25°	59.07°
	Elevation at B	90.00°	90.00°	90.00°	90.00°	90.00°	90.00°
	Elevation at C	6.16°	12.70°	18.91°	24.69°	29.97°	34.74°
	Elevation at D	8.36°	16.92°	24.77°	31.72°	37.77°	42.96°
	Elevation at E	7.69°	15.54°	22.83°	29.40°	35.22°	40.30°

Table 2 - PDOP and elevations simulating 5 combination of 4 fixed bases and 1 combination of 5 fixed bases for each repeater

3. PDOP simulations with 12 fixed bases

Other simulations were done with 12 fixed bases are show in Table 3 and illustrated in the Figure 3. The results are shown in Table 4.

City	Element	Latitude	Longitude	Altitude
Rio Claro	Base A	-22° 24' 48"	-47° 34' 11"	592 m
Piracicaba	Base B	-22° 43' 30"	-47° 38' 51"	524 m
Botucatu	Base C	-22° 53' 25"	-48° 27' 19"	828 m
Campinas	Base D	-22° 54' 23"	-47° 03' 42"	677 m
Ita	Base E	-23° 15' 57"	-47° 17' 57"	577 m
Ita	Base F	-22° 17' 47"	-48° 33' 28"	522 m
Itotuba	Base G	-22° 17' 03"	-48° 07' 36"	647 m
Santa Maria da Serra	Base H	-22° 34' 02"	-48° 09' 38"	495 m
Catubas	Base I	-23° 00' 55"	-48° 00' 38"	503 m
Botuca	Base J	-23° 17' 00"	-47° 40' 20"	637 m
Mogi Mirim	Base K	-22° 25' 55"	-46° 57' 30"	617 m
Bofete	Base L	-23° 06' 08"	-48° 15' 28"	576 m
Piracicaba	Repeater R1	-22° 43' 30"	-47° 38' 51"	variable
Limeira	Repeater R2	-22° 33' 52"	-47° 24' 01"	variable
Chaquoadela	Repeater R3	-22° 30' 39"	-47° 46' 40"	variable
Antônio	Repeater R4	-22° 47' 22"	-48° 07' 38"	variable

Table 3 - Latitude, longitude and altitude of 12 bases and 4 repeaters

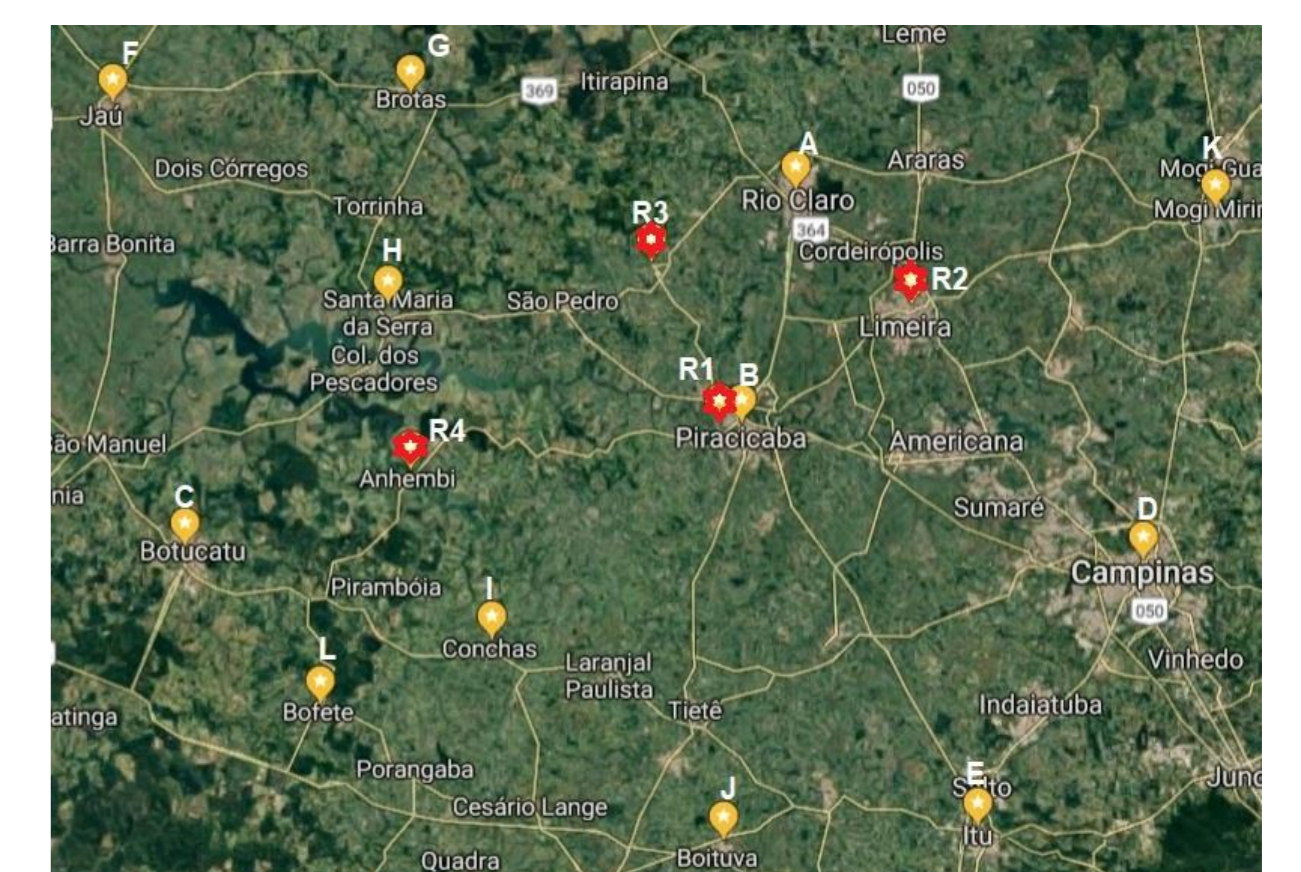


Figure 3 – position of the 12 bases and the 4 repeaters (adapted from Google Maps)

Repeater	Altitude (m)	PDOP					Average elevation
		10000	20000	30000	40000	50000	
Repeater R1	PDOP	1.38	1.56	1.75	1.96	2.20	2.48
	Elevation at A	14.86°	28.68°	39.63°	47.95°	54.25°	59.07°
	Elevation at B	90.00°	90.00°	90.00°	90.00°	90.00°	90.00°
	Elevation at C	6.16°	12.70°	18.91°	24.69°	29.97°	34.74°
	Elevation at D	8.36°	16.92°	24.77°	31.72°	37.77°	42.96°
	Elevation at E	7.69°	15.54°	22.83°	29.40°	35.22°	40.30°
	Elevation at F	5.58°	10.49°	15.68°	20.55°	25.14°	29.41°
	Elevation at G	7.67°	15.56°	22.88°	29.48°	35.32°	40.41°
	Elevation at H	9.70°	19.32°	27.92°	35.33°	41.59°	46.83°
	Elevation at I	10.91°	21.58°	30.88°	38.69°	45.05°	50.27°
	Elevation at J	8.60°	17.35°	25.33°	32.38°	38.47°	43.67°
	Elevation at K	6.80°	13.94°	20.67°	26.73°	32.25°	37.17°
Elevation at L	5.19°	10.44°	15.31°	20.57°	25.70°	30.16°	
Ave. elevation	9.46°	18.95°	26.61°	33.22°	39.11°	42.09°	
Repeater R2	PDOP	3.87	2.44	2.23	2.30	2.51	2.78
	Elevation at A	14.86°	28.68°	39.63°	47.95°	54.25°	59.07°
	Elevation at B	16.98°	32.09°	43.48°	51.76°	57.82°	62.35°
	Elevation at C	4.59°	9.51°	14.29°	18.87°	23.21°	27.27°
	Elevation at D	10.27°	20.57°	29.64°	37.32°	43.70°	48.95°
	Elevation at E	6.85°	13.99°	20.53°	26.67°	32.14°	37.04°
	Elevation at F	4.41°	9.00°	13.47°	17.77°	21.87°	25.74°
	Elevation at G	6.58°	13.42°	19.88°	25.85°	31.26°	36.11°
	Elevation at H	6.93°	13.98°	20.63°	26.73°	32.24°	37.14°
	Elevation at I	6.75°	13.66°	20.17°	26.17°	31.61°	36.47°
	Elevation at J	6.33°	12.91°	19.15°	24.94°	30.24°	35.01°
	Elevation at K	11.10°	22.06°	31.53°	39.41°	45.83°	51.05°
Elevation at L	5.98°	10.34°	15.44°	20.29°	24.65°	29.10°	
Ave. elevation	8.93°	17.51°	24.89°	31.88°	36.55°	41.17°	
Repeater R3	PDOP	3.66	2.30	2.07	2.12	2.28	2.51
	Elevation at A	21.40°	38.94°	50.74°	58.60°	64.02°	67.92°
	Elevation at B	19.17°	35.52°	47.19°	55.31°	61.07°	65.29°
	Elevation at C	6.43°	13.25°	19.69°	25.65°	31.06°	35.92°
	Elevation at D	6.21°	12.70°	18.86°	24.69°	29.85°	34.59°
	Elevation at E	5.58°	11.31°	16.84°	22.06°	26.92°	31.38°
	Elevation at F	6.45°	13.07°	19.35°	25.17°	30.47°	35.25°
	Elevation at G	12.04°	23.79°	33.75°	41.83°	48.28°	53.43°
	Elevation at H	13.40°	26.03°	36.44°	44.65°	51.04°	56.05°
	Elevation at I	8.88°	17.76°	25.84°	32.94°	39.06°	44.26°
	Elevation at J	6.19°	12.64°	18.76°	24.47°	29.69°	34.42°
	Elevation at K	6.31°	12.86°	19.07°	24.85°	30.12°	34.88°
Elevation at L	6.58°	13.31°	19.76°	25.62°	30.99°	35.81°	
Ave. elevation	9.88°	19.27°	27.19°	33.81°	39.38°	44.10°	
Repeater R4	PDOP	3.95	2.51	2.29	2.35	2.53	2.78
	Elevation at A	7.56°	15.29°	22.49°	29.00°	34.78°	39.84°
	Elevation at B	10.77°	21.33°	30.57°	38.32°	44.71°	49.94°
	Elevation at C	14.48°	28.38°	39.36°	47.24°	54.08°	58.94°
	Elevation at D	4.83°	9.44°	14.88°	19.59°	24.04°	28.20°
	Elevation at E	5.38°	10.98°	16.37°	21.47°	26.22°	30.62°
	Elevation at F	7.67°	15.46°	22.70°	29.24°	35.03°	40.10°
	Elevation at G	9.48°	19.05°	27.62°	35.03°	41.30°	46.55°
	Elevation at H	20.92°	38.09°	49.83°	57.75°	63.26°	67.24°
	Elevation at I	18.89°	35.07°	46.70°	54.85°	60.65°	64.91°
	Elevation at J						